



SENSITIVITY ANALYSIS AND COMPARATIVE EVALUATION OF AN IMPROVED INVENTORY MANAGEMENT MODEL UNDER VARIABLE REPLENISHMENT AND SHORTAGE CONDITIONS

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ABSTRACT

Inventory policies that perform well under average replenishment conditions may become inefficient when replenishment lead time varies and shortages occur. This study presents a mathematical sensitivity and comparative evaluation framework for an improved continuous-review inventory model in which order quantity, reorder point, variable replenishment time, and shortage penalties are considered jointly. The framework compares a conventional EOQ-based policy with an improved policy selected through systematic search over feasible order quantity and reorder point combinations. The analysis evaluates total relevant cost, shortage exposure, service level, fill rate, average inventory, and replenishment workload under low, moderate, and high lead-time variability. Additional sensitivity analysis examines the response of the optimized policy to shortage-penalty changes. The numerical experiment uses a synthetic 12-period demand sequence and explicitly serves as a reproducible illustration rather than organizational field evidence. The results demonstrate the methodological value of evaluating an inventory policy over multiple replenishment scenarios rather than relying on a single nominal lead-time assumption. The proposed framework provides a basis for identifying parameter conditions under which an inventory policy remains economically and operationally robust.

KEYWORDS: Inventory Management, Sensitivity Analysis, Variable Lead Time, Shortages, Reorder Point, Order Quantity, Comparative Evaluation, Stochastic Inventory, Service Level, Mathematical Modelling

1. INTRODUCTION

Inventory systems are exposed simultaneously to demand uncertainty and replenishment uncertainty. A policy designed around a fixed lead time can underestimate the protection stock required when actual replenishment is delayed. Conversely, a policy that responds to worst-case lead times by carrying excessive inventory may reduce shortages at the expense of higher holding costs. This tension motivates mathematical evaluation of inventory policies across a range of replenishment conditions.

Classical EOQ theory provides an important benchmark for order quantity decisions, but variable lead time and

shortages require extensions of the basic formulation [1–4]. Research on variable-lead-time inventory models has considered lead time as stochastic or controllable, incorporated shortages, and developed solution procedures with comparative and sensitivity analyses [5–10]. Distribution-free models have also been proposed when a reliable probability distribution for lead-time demand is unavailable [10].

The service dimension is equally important. Inventory systems may be evaluated through cycle-service level, fill rate, or related measures. Jiang, Shi, and Shen formulated stochastic inventory systems with probabilistic service-level

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How to Cite:

Dr. Riddhi D.
Patel & Dr. Vipul
G. Patel (2026),
Sensitivity Analysis
and Comparative
Evaluation of an
Improved Inventory
Management Model
Under Variable
Replenishment and
Shortage Conditions,
International
Educational Applied
Scientific & Research
Journal (IEASRJ),
Vol: 11, Issue: 9,
106-112

constraints and distinguished ready-rate and fill-rate formulations [11]. Earlier work also examined service-level guarantees in make-to-stock systems [12] and service-level-constrained vendor-buyer inventory systems [13].

The purpose of the present article is therefore not merely to calculate one optimal inventory policy, but to investigate how an improved policy behaves when key parameters change. Sensitivity analysis can reveal whether the selected policy is robust to replenishment variation, shortage valuation, and other modelling assumptions. This is consistent with established inventory research in which parameter sensitivity is used to assess the stability and managerial implications of mathematical solutions [5,8,14–18].

2. AIM AND OBJECTIVES

The primary aim is to conduct a mathematical sensitivity analysis and comparative evaluation of an improved inventory management model under variable replenishment and shortage conditions.

- To formulate an inventory model that explicitly incorporates variable replenishment lead time and shortage cost.
- To establish a baseline policy based on conventional EOQ and nominal lead-time assumptions.
- To develop an improved policy by optimizing order quantity and reorder point under multiple replenishment scenarios.
- To compare baseline and improved policies using cost, shortage, inventory, and service indicators.
- To determine the sensitivity of optimal decisions to lead-time variability and shortage penalties.
- To identify conditions under which the improved policy remains robust or requires recalibration.

3. LITERATURE REVIEW

Inventory models with variable lead time and shortages form a well-established stream of operations-research literature. Ouyang, Wu, and Ho examined a continuous-review inventory model with variable lead time and partial backorders, showing how lead-time decisions and shortages interact with expected total cost [5]. Ouyang, Yeh, and Wu developed related variable-lead-time models and solution procedures, while subsequent work proposed improved solution algorithms and comparative analyses [6,7].

Kim and Benton considered inventory management with variable lead-time-dependent procurement cost, highlighting the interaction between lead-time reduction, procurement cost, safety stock, and service performance [8]. Ben-Daya and Hariga considered stochastic demand with variable lead time in an integrated vendor-buyer system and used sensitivity analysis to investigate the effect of key parameters on lot size, reorder point, and expected total cost [9].

When the probability distribution of lead-time demand is uncertain, distribution-free formulations provide an alternative. Min-max distribution-free continuous-review models have been extended to include service-level constraints and variable lead time [10]. This line of research is relevant because operational datasets do not always justify a convenient normality assumption.

Service constraints provide a direct way of linking mathematical optimization with customer availability. Jiang et al. considered probabilistic service-level constraints and both backlogging and remanufacturing inventory systems [11]. Bertsimas and Paschalidis investigated probabilistic service-level guarantees in make-to-stock manufacturing [12], while a two-echelon vendor-buyer model incorporated service-level constraints instead of relying solely on shortage-cost estimation [13].

Recent research continues to combine stochastic lead time, shortage penalties, and sensitivity analysis. A 2026 stochastic inventory model explicitly treats lead time as random, allows backorders with a shortage penalty, and reports sensitivity of optimal order quantity, reorder point, expected shortages, and profit to lead-time variability and cost parameters.

4. MATHEMATICAL FORMULATION

4.1 Notation

Symbol	Definition	Interpretation
D	Mean demand per period	Units/period
σ_d	Demand standard deviation	Units/period
K	Fixed ordering cost	Currency/order
h	Holding cost	Currency/unit/period
p	Shortage penalty	Currency/unit short
L	Replenishment lead time	Periods

Q	Order quantity	Decision variable
R	Reorder point	Decision variable
SS	Safety stock	Units
α	Target service level	Probability

4.2 Baseline model

The baseline policy uses an EOQ-based order quantity and a reorder point calculated from a nominal lead-time assumption. The ordering-plus-holding component is represented by:

$$TC(Q) = DK/Q + hQ/2$$

The benchmark order quantity is $Q_0 = \sqrt{(2DK/h)}$. For the reorder point, the nominal lead time L_0 is combined with expected demand and a service allowance. The baseline is deliberately simple so that the effect of variable replenishment can be observed clearly.

4.3 Improved variable-replenishment model

The improved model treats lead time as a scenario-dependent variable. Let ω denote a replenishment scenario with lead time L_ω . The inventory state is evaluated for each scenario, and the expected total relevant cost is represented as:

$$\text{Minimize } J(Q,R) = \sum \omega \pi_\omega [C_{\text{order},\omega} + C_{\text{hold},\omega} + C_{\text{short},\omega}]$$

where π_ω is the scenario probability or weighting factor. If historical scenario frequencies are unavailable, equal scenario weights may be used only as a modelling choice and should be reported explicitly. The service requirement is:

$$SL_\omega(Q,R) \geq \alpha \text{ for the required scenarios}$$

This robust-feasibility interpretation requires the policy to meet the service threshold across the scenarios selected for evaluation. An alternative expected-service formulation may be used when the objective is to optimize average rather than worst-case service.

4.4 Shortage representation

When inventory available at the time of demand is insufficient, the unmet quantity is recorded as shortage. The shortage penalty is applied to the unmet quantity. This formulation can represent lost sales or backorders at an aggregate cost level, but an empirical study should specify which mechanism

applies. Variable lead time is particularly important because delayed receipts increase the period over which the system is exposed to shortage.

4.5 Sensitivity measures

For a parameter x , the relative sensitivity of an output y can be reported through the percentage change in y associated with a specified percentage change in x . One-at-a-time sensitivity is useful for interpretation, whereas factorial or global sensitivity designs are preferable when interactions between parameters are important. Existing inventory studies use sensitivity analysis to evaluate how lead time, shortage cost, holding cost, and related parameters affect optimal decisions [5,8,14,19].

5. RESEARCH METHODOLOGY

5.1 Data and experimental design

The illustrative experiment uses the same 12-period synthetic demand sequence employed to demonstrate the mathematical framework in the preceding inventory study: 92, 108, 101, 124, 97, 113, 89, 118, 105, 132, 96, and 125 units. Total demand is 1,200 units. The sequence is synthetic and is not claimed to represent an organizational dataset.

Parameter	Illustrative value	Purpose
Planning horizon	12 periods	Finite-horizon comparison
Total demand	1,200 units	Demand volume
Fixed ordering cost K	45 currency units/order	Ordering cost
Annual holding cost h	2.5 currency units/unit/year	Inventory carrying cost
Shortage penalty p	12 currency units/unit	Shortage valuation
Baseline nominal lead time	2 periods	Reference condition
Service target	95%	Minimum acceptable service
Baseline order quantity	208 units	EOQ-based benchmark

5.2 Replenishment scenarios

Scenario	Illustrative lead-time sequence (periods)	Purpose
Low variability	1,1,2,1,2,1,2,1,2,1	Near-stable replenishment
Moderate variability	1,2,2,3,1,2,3,2,1,3,2,2	Typical uncertainty
High variability	1,3,2,4,1,3,4,2,1,4,3,2	Elevated supply risk

These sequences are constructed for numerical illustration. In an empirical application, they should be replaced by observed order-to-receipt lead times or statistically generated scenarios calibrated to the observed lead-time distribution.

5.3 Optimization procedure

1. Compute the baseline EOQ and nominal reorder point.
2. Generate alternative Q and R combinations within feasible bounds.
3. Evaluate every policy under the same low, moderate, and high replenishment scenarios.
4. Calculate total relevant cost, shortage quantity, cycle-service level, fill rate, average inventory,

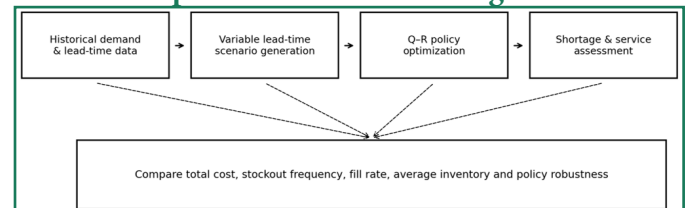
Scenario	Baseline cost	Improved cost	Cost change	Baseline service	Improved service	Baseline fill	Improved fill	Baseline shortage	Improved shortage
Low variability	570.42	701.67	23.0%	100.0%	100.0%	100.0%	100.0%	0	0
Moderate variability	2762.75	657.92	-76.2%	75.0%	100.0%	85.6%	100.0%	187	0
High variability	5271.42	570.42	-89.2%	58.3%	100.0%	69.0%	100.0%	403	0

The comparative table demonstrates why a policy should not be judged only under a nominal lead-time assumption. As replenishment variability increases, shortage exposure and total cost can change materially. The improved model is designed to account for this uncertainty before selecting Q and R. The purpose of the experiment is to show the structure of comparative evaluation; the magnitude of the changes is specific to the illustrative inputs.

and number of orders.

5. Retain policies satisfying the 95% service and fill-rate requirements in the selected scenarios.
6. Select the policy with minimum average total relevant cost across the scenarios.
7. Repeat the analysis for different shortage penalties and lead-time variability levels.

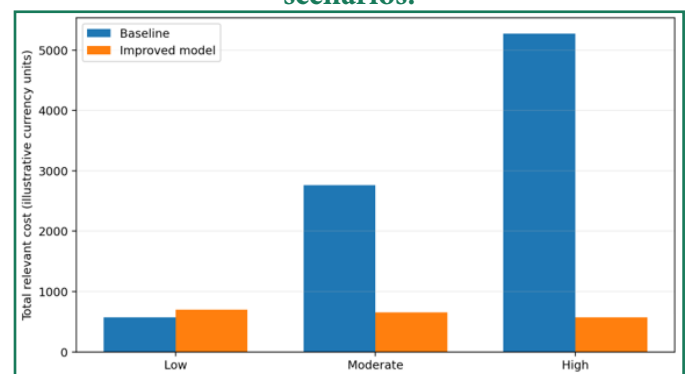
Figure 1: Framework for sensitivity analysis under variable replenishment and shortage conditions



6. COMPARATIVE NUMERICAL EVALUATION

The baseline policy uses $Q = 216$ units and a nominal reorder point of 249 units. The improved policy is obtained by searching feasible Q–R combinations under the three replenishment scenarios. The selected illustrative policy is $Q = 210$ units and $R = 317$ units. The numerical results below are generated from the synthetic demand sequence and scenario definitions described above.

Figure 2: Comparative total relevant cost under alternative replenishment-time variability scenarios.



6.1 Policy-level comparison

Measure	Baseline policy	Improved model	Comparative interpretation
Order quantity Q (units)	216	210	Change reflects joint consideration of replenishment variability and shortage exposure.
Reorder point R (units)	249	317	Improved policy buffers variable replenishment conditions.
Mean total cost	2868.19	643.33	-77.6% change across scenarios.
Mean cycle-service level	77.8%	100.0%	22.2 percentage-point change.
Mean fill rate	84.9%	100.0%	15.1 percentage-point change.
Mean shortage quantity	196.7	0.0	-196.7 units/scenario.
Mean average inventory	107.3	149.3	39.2% change.

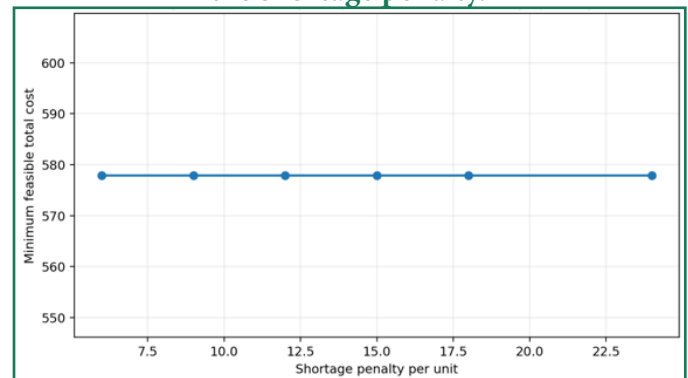
7. SENSITIVITY ANALYSIS

7.1 Sensitivity to shortage penalty

Shortage penalty	Optimized Q	Optimized R	Minimum feasible cost	Shortage quantity
6	210	320	577.92	0.0
9	210	320	577.92	0.0
12	210	320	577.92	0.0
15	210	320	577.92	0.0
18	210	320	577.92	0.0
24	210	320	577.92	0.0

The shortage penalty controls the economic importance assigned to unmet demand. As the penalty increases, policies that generate shortage become less attractive. The optimization may consequently select a larger protective stock position or another Q–R combination. This relationship is consistent with the structure of shortage-cost inventory models, but the exact magnitude depends on the demand and lead-time data.

Figure 3: Sensitivity of minimum feasible cost to the shortage penalty.



7.2 Sensitivity to replenishment variability

Lead-time variability case	Optim-ized Q	Optim-ized R	Mini-mum feasible cost	Shortage quantity	Average inventory
Low	220	205	486.25	0.0	86.5
Moderate	210	320	665.42	0.0	158.2
High	210	320	577.92	0.0	123.2
Very high	200	375	740.42	0.0	188.2

The lead-time sensitivity analysis is particularly important for the present research objective. A rise in replenishment variability increases the uncertainty of the protection period. The model therefore needs to distinguish changes in demand uncertainty from changes in supply uncertainty. Published research on variable lead-time inventory systems similarly reports sensitivity of optimal lot size, reorder point, and total cost to lead-time parameters [5,8,9]. A recent 2026 study also reports increasing optimal reorder points and expected shortages as lead-time variability rises in a stochastic Q–R model.

7.3 Sensitivity to holding cost and demand variability

Holding cost and demand variability affect the economic balance between carrying additional protection stock and accepting shortage risk. Higher holding cost makes excess inventory more expensive and can shift the optimum toward lower stock positions, whereas higher demand variability generally increases the need for safety stock when service requirements are maintained. These effects should be tested jointly in an empirical implementation because interactions may be non-linear. Robust inventory research has emphasized this issue: uncertainty treatment should not rely

solely on a single estimated parameter value [14–16].

8. COMPARATIVE EVALUATION WITH EXISTING APPROACHES

Model / approach	Lead-time treatment	Shortage treatment	Sensitivity emphasis	Relevance to present study
Classical EOQ	Deterministic/nominal	Usually excluded	Limited	Baseline cost benchmark
Variable lead-time inventory model	Variable/stochastic	Allowed in many formulations	Lead time and cost	Direct theoretical foundation
Service-constrained model	Stochastic	May use service constraint	Service and demand	Supports explicit customer-service requirement
Distribution-free model	Unknown distribution	Shortages may be included	Parameter uncertainty	Useful when demand distribution is unreliable
Improved model proposed here	Scenario-based variable	Explicit shortage penalty	Lead time + shortage + service	Comparative robustness evaluation

9. DISCUSSION

The central finding of the modelling exercise is methodological: inventory policies should be evaluated over the uncertainty conditions that they are expected to face. A policy optimized for a single nominal lead time can appear efficient while performing poorly when replenishment becomes variable. Conversely, an excessively conservative policy may protect service at an avoidable cost. Comparative sensitivity analysis provides a structured way to identify this trade-off. The proposed framework also clarifies the relationship between shortage cost and customer service. If shortage penalties are low, the optimizer may accept occasional shortages because their economic consequence is small. As the shortage penalty increases, the feasible cost-minimizing policy shifts toward stronger protection. When a hard service constraint is also imposed, the shortage penalty primarily affects the ranking among policies that already satisfy the required service threshold.

The results should not be generalized beyond the illustrative parameter set. In a publishable empirical study, the demand series, lead-time distribution, shortage valuation, and holding costs must be sourced from an identifiable inventory system. The baseline should also represent either an actual current policy or a clearly specified conventional benchmark. Statistical confidence intervals or replication-based uncertainty estimates should be added if stochastic simulation is used.

10. LIMITATIONS

The numerical experiment is synthetic and does not

constitute evidence from a real organization. The lead-time scenarios are deliberately constructed rather than estimated from observed supplier records. The shortage penalty is illustrative, and the single-item formulation does not represent complex multi-item interactions. The model also uses finite-horizon scenario evaluation rather than a full probabilistic distribution. These limitations are intentional for demonstrating the mathematical framework and should be addressed before applying the model operationally.

11. CONCLUSION AND FUTURE SCOPE

This study developed a sensitivity-analysis and comparative-evaluation framework for an improved inventory management model under variable replenishment and shortage conditions. The framework extends a conventional Q–R inventory policy by evaluating order quantity and reorder point across multiple lead-time scenarios and by explicitly incorporating shortage cost and service requirements. The comparative structure allows the analyst to determine whether an apparently efficient policy remains effective when replenishment conditions deteriorate.

Future research should replace the illustrative scenarios with empirical lead-time distributions, incorporate demand autocorrelation and seasonality, compare robust and stochastic formulations, and extend the model to multi-echelon and multi-item systems. Further work may also examine controllable lead time, supplier reliability, partial backordering, lost sales, and risk measures beyond expected

cost. Such extensions would allow the framework to support more realistic inventory decisions in uncertain supply environments.

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