



USING ITERATIVE INQUIRY SKILLS TO SOLVE TRIGONOMETRY EQUATIONS

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ABSTRACT

Instruction should not focus on transmitting plans to the learner but rather on developing the skills of the learner to construct (or reconstruct) plans in response to situational demands and opportunities. Plans must be constructed, tested, and revised as a function of the particular encounters in the environment. I conducted a case study to help a group of class 10 students solve a variety of trigonometric equations using iterative inquiry approach. I found this method effective. Students were given a base trigonometric equation to solve. They were guided to visualize a series of problems through the base problem. Students could see the internal structure and pattern in different problems; they could relate to the base problem. They could display cognitive flexibility through multiple representations. They were encouraged to form new trigonometric equations by modifying base problem. This way they could discover lot of similarity amongst the variety of trigonometric equations given in their textbook.

KEY WORDS: multiple representations, cognitive flexibility, iterative inquiry approach.

INTRODUCTION:

Duffy and Jonassen (1992) argue that instruction should not focus on transmitting plans to the learner but rather on developing the skills of the learner to construct (or reconstruct) plans in response to situational demands and opportunities. Thus, instruction should provide contexts and assistance that will aid the individual in making sense of the environment as it is encountered. Plans must be constructed, tested, and revised as a function of the particular encounters in the environment. Any propositionalization is simply one representation of prior experience. The individual, can reconceptualize, reconstruct, and repropotionalize that experience in many different ways. Home work is not about finding answers, which are all in the back of text book or guide book. It's all about recognizing when you really don't understand and need a little help (Marlo Warburton). But unless in school mathematics time is greatly expanded at the expense of other subjects, the extra practice and self assessing of home work remain critical parts of mastering arithmetic skills, particularly for students with relatively weak mathematics backgrounds. So it is essential that maths teachers design home work systems that encourage the kinds of behaviors and attitudes that lead to long run success in mathematics. If they fail, the student for whom home work is most essential will be those least likely to complete it. To make a home work system productive for all students, it must encourage mathematical thinking and skill development; provide useful, non judgmental feedback; enhance student responsibility for learning; help students self assess without advertising weakness; and promote student self confidence and motivation for learning (Vatterott. 2009, p.94).

OBJECTIVES:

Many misconceptions that prohibit students from developing a deeper understanding of trigonometry have occurred and persisted since their early learning experiences. For example, misconception regarding the generalizability of the trigonometric equations is a challenge. Most important, traditional instruction even when designed to address misconception, often does not provide for sufficient conceptual change in student understanding (Vlassis, 2004). For students to be successful in trigonometry, they must have a truly conceptual understanding of key trigonometric equations, algebraic features as well as the procedural skills to complete a problem. One strategy to correct students' misconception combines the use of worked example problem in the classroom with student self explanation. Self explanation is the "activity of generating explanations to oneself." (Chi 2000, p.164), especially "in attempt to make sense of new information" (p.163) as one reads or studies.

Forming questions as another mode of streamlining creativity can be part of formative as well as summative processes. When students start to make connections among graphical, tabular, symbolic, and descriptive contents for functional relationships, all representations of the same relationship- they may feel inspired and engaged by seeing structure and beauty among these representations (Middleton and Jansen, 2011). Iterative inquiry approach helps student to see the internal structure and modify it to form a new situation. Conversely a seemingly new situation can be analyzed to familiar problem. Helping students to be creator of intelligently designed trigonometric equation is the purpose.

A section of 34 students of Class 10th of CBSE were allowed to modify the questions related to trigonometric identities. In their prescribed text book four examples and ten problems has been provided for practice. Students feel that they need to practise more related problems looking at the recent assessment trend. Most of

the questions are twisted. Through this activity students learn how to frame new trigonometric equations related questions from a given question. They look for multiple representations of the same problem. In this process they end up with discovering patterns leading to a group of questions. To help students search a base problem and look for other problems that fit this pattern. To know if students can better handle twisted problems set in their standardized summative assessments after going through this exercise of question generation.

METHODOLOGY:

A worked example problem to be differentiated from working an example problem shows students in already completed problem and directs their attention to certain steps of the task as the focus of questioning. Self explanation, then, specifically encourages students to identify the reasoning behind the steps that they see carried out and to explain why these steps were completed. This strategy of providing worked example problems coupled with prompts for self-explanation has recently been shown to influence students' learning positively in both traditional (Booth, Koedinger, and Pare-Blagoev 2011) and computer based classrooms (Booth et al. 2013). The technique has prominence in classroom settings (Ward & SWeller 1990; Schwonke et al. 2009).

To properly interpret question, it is probably useful to be involved in some group discussion and to learn by trial and error. While solving one example of trigonometric identity from class 10th NCERT textbook teacher may initiate the discussion by asking the students to go through the solution of the problem and understand the procedure; whole class discussion continued this way; it was primarily led by students, with our facilitation. By using probing questions that require students to explain a previously worked example, teacher can ensure that students are making sense of what proving a trigonometric equation really entails. Student also engages in reasoning while constructing explanations and strengthens critical thinking skills while critiquing the correct or incorrect solution.

RESULTS:

Teacher thinks to use the internal structure of mathematics itself as a source of motivation, a safeguard against confusion, a protection against forgetting, an alternative to un mindful drill. Indeed, listening to the child's suggestions appears to us be one of the cornerstone upon which "new mathematics" is built. Do not require the child to read your mind; do not require him to "do it your way" - show a simple respect for the child's intellectual, analytical, and problem solving autonomy and you will discover that he is much cleverer child than you had ever imagined! (New Trends in Mathematics Teaching, Vol.II, 1970. Unesco.). As Warwick Sawyer has put it, "the poor teacher asks the child: "Don't you remember the rule?" where as the good teacher asks "Don't you see the pattern?"

Teacher initiates the discussion by asking how can I form more questions from the first question. A seminar type discussion goes and teacher initially helps to frame questions. Students generate a group of eight questions from the first question.

I Group of Equations:

1. Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ (Example no. 15)
2. Prove that $\frac{\sin \theta + \cos \theta - 1}{\sin \theta - \cos \theta + 1} = \sec \theta - \tan \theta$ (Reciprocal of Q.1)

3. $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \sec \theta + \tan \theta$ (modifying RHS of Q.1, 1 is recognized as $\sec^2 \theta - \tan^2 \theta = (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)$. Numerator of RHS of Q.1. replaced with this turns the RHS to be $\frac{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)}$. Students come forward with suggestions and necessary modification.)

4. $\frac{\sin \theta + \cos \theta - 1}{\sin \theta - \cos \theta + 1} = \frac{1}{\sec \theta + \tan \theta}$ [Reciprocal of Q.3. It is also recognized as Q.2 with modified RHS. i.e. denominator 1 of Q.2 replaced by $\sec^2 \theta - \tan^2 \theta = (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)$]

5. Prove that $\frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \frac{1}{\csc \theta - \cot \theta}$ (Complementary of Q.1)

$$\frac{\cos \theta + \sin \theta - 1}{\cos \theta - \sin \theta + 1} = \csc \theta - \cot \theta \quad (\text{Reciprocal of Q.4 or complementary of Q.2})$$

6. Prove that $\frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \csc \theta + \cot \theta$ [Q.5 (V) Exercise 10.4 of NCERT text book-2012. It is nothing but Q.4 with modified RHS.]

7. Prove that $\frac{\cos \theta + \sin \theta - 1}{\cos \theta - \sin \theta + 1} = \frac{1}{\csc \theta + \cot \theta}$ (Reciprocal of Q.6)

II Group of Equations

1. $(\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$ (Q.5-i, Exercise 10.4)

2. $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$ (Q.5-vi, Exercise 10.4)

Student response for generating similar questions from these two similar questions

3. $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \csc \theta - \cot \theta$ (Square root taken on both sides of Q.1)

4. $\frac{\cos \theta + \sin \theta - 1}{\cos \theta - \sin \theta + 1} = \csc \theta - \cot \theta$ (Q.5 of I group of equations)

5. Combining Q.3 and Q.4 one gets $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\cos \theta + \sin \theta - 1}{\cos \theta - \sin \theta + 1}$

6. $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1}$ (Reciprocal of Q.5)

7. $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \frac{1}{\csc \theta - \cot \theta}$ (Reciprocal of Q.3)

8. $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \csc \theta + \cot \theta$ (modifying RHS of Q.7)

9. $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{1}{\csc \theta + \cot \theta}$ (Reciprocal of Q.10)

10. $\frac{\cos \theta + \sin \theta - 1}{\cos \theta - \sin \theta + 1} = \frac{1}{\csc \theta + \cot \theta}$

11. $\frac{1 + \cos \theta}{1 - \cos \theta} = (\csc \theta + \cot \theta)^2$ (similar to Q.2 obtained by squaring Q.10 or replacing Q.2 by their complementary angle)

12. $(\sec A + \tan A)^2 = \frac{1 + \sin A}{1 - \sin A}$ (squaring Q.2 or similar to Q.9)

13. Replacing Q.1 with complementary angles $(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$

14. Taking square root on both sides, $\sec A - \tan A = \sqrt{\frac{1 - \sin A}{1 + \sin A}}$

15. Taking the reciprocal of both sides $\frac{1}{\sec A - \tan A} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$

16. Comparing with Q.1 of group I, $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ (Example no. 15)

17. $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$

The reasoning was part of the mathematical concept of the lesson. The fact that the explanations were coming from peers rather than solely from the teacher seemed to give them more merit in the eyes of the students. Finally I was reminded of the importance of multiple representations. Some students understood the mathematical concept with just the pattern involved. Others had an 'aha' moment when they could predict twisted questions and probable solution procedure orally, whereas still others truly grasped the concept when they saw one question belonging to a family of questions having same solution procedure. Having all this diverse but symmetrical representation seemed to give students a much more complete idea of what it means to prove a trigonometric equation. These tasks served to promote deeper understanding of proving trigonometric equations, which will help students of all ability levels prepare for higher level mathematics. I also know that effective mathematics lessons consists of tasks that are "cognitively demanding as well as pedagogical practices that are suitable to support collaboration and discourse among students, and thoughtful engagement with mathematical reasoning, problem solving, and explanation" (Silver 2010, p.1). Observing our students grow and gain understanding throughout this unit of study was exciting. The connections that students made among the problems to prove trigonometric equations enabled them to make their own decisions when starting a trigonometric equation proof they had never seen before. They were able to become proof doers, in part because the abstraction within the problems was no longer intimidating- it had a pattern!

DISCUSSION:

They could manipulate the same procedure to a big family of problems connected through some hidden pattern. Students could better handle twisted problems set in their standardized summative assessments. They were promptly visualizing the twisted problems and translating it as the multiple representation of some base problem whose solution procedure is well known to them. This activity boosted their self confidence. They showed increasing interest at discovering more problems connected through some pattern. Helping students to twist the problems empowered them. Viewing new problems as extension of present understanding with the help of patterns streamlined their efforts through repetition. This process was made interesting by allowing students to form groups. Each group challenged the other group by providing difficult twist. In some occasions disproportionate twist was given to different section of one problem; students were then required to identify the correct version of the new question. Open question like state all possible multiple representations of the base question provides mental challenge to creative answers. With this they are able to visualize the 14 problems as multiple representations of few problems. They become confident that solving one question is equivalent to solving other question of that group. They do not view each question separately; rather they view it as part of bigger family of problems. They understand that with little modification the differently looking problem can be made to look like the problem with which they are already habituated. This helps them to establish correlation between different problems.

Students know where to start and where to end, so they try promising mathematics paths until one works. No one snickers when a student takes a wrong turn or retraces steps. Feedback is prompt and private- gone is the fear that tomorrow's home work check will reveal an embarrassing misstep in a student's thinking. Not being able to figure something out is an expected part of learning; instead of hoping that no one will notice, students can take the initiative and ask for help. Once students learn each successful problem framing approach, they earn full credit for that learning. It's a friendly place.

Unfortunately, analyzing question is one of those skills that does not lend itself to standardized multiple choice testing. A certain amount of comprehension is going to be testable, but higher level critical thinking is probably best judged on a subjective scale. Cobb (1944) contrasts the approach of delivering mathematics as "content" against the technique of fostering the emergence of mathematical ideas from the collective practices of the classroom community. Emphasis is growing on the teacher's use of multiple epistemologies, to maintain dialectic tension between teacher guidance and student-initiated exploration, as well as between social learning and individual learning. Rather than thinking of truth in terms of reality, von Glasersfeld focuses instead on the notion of viability: "to the constructivist, concepts, models, theories, and so on are viable if they prove adequate in the contexts in which they were created." Constructivism views that

knowledge is not 'about' the world, but rather 'constitutive' of the world (Scherman, 1995). Vico maintained that "the human mind can only know what the mind has made." i.e. human can understand only what they have themselves constructed. According to McKenzie (1999), "the defining features of successful scaffolding include clear direction, purpose, and expectation. Scaffolding requires continuous sorting and sifting as part of a 'puzzling' process- the combining of new information with previous understandings to construct new ones. Students are adding on, extending, refining and elaborating. It is almost as if they are building a bridge from their preconceptions to a deeper, wiser, more astute view of whatever truth matters for the question or issue at hand." There are different ways to scaffold instruction for students. According to Bransford, Brown and Cocking (2000), some educators favor an apprenticeship model whereby an expert models an activity, provides the learner with advice and examples, guides the student in practice and then tapers off support until the student can do the task alone; others prefer method that encourage ongoing use of tools and consultation with other people, arguing that in real life few people ever work exclusively on their own. Most agree that scaffolding is particularly more effective in areas in which students need to be more self-reliant (Banaszynski, 2000).

According to most organismic perspectives, knowledge systems develop by means of recurrent qualitative shifts in the direction of increased complicity (Werner, 1957). Thus, knowledge can never be considered an accurate depiction of reality; since each new refinement will require justification at a newer and higher level. Vygotsky (1978) advocates the concept that learning occurs in the "zone of proximal development", which is defined as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers". In fact, question making taps into and triggers the students' innate curiosity about the world and how things work. Students do not reinvent the wheel but, rather, attempt to understand how it turns, how it functions. They become engaged by applying their existing knowledge and real world experience, learning to hypothesize, testing their theories, and ultimately drawing conclusions from their findings (Siegel, 2004). In the constructivist model, the students are urged to be actively involved in their own process of learning. The teacher functions more as a facilitator who coaches, mediates, prompts, and helps students develop and assess their understanding, thereby their learning. One of the teacher's biggest jobs becomes asking good questions (Seigel, 2004). Rather, teachers act as "guides on the side" who provide students with opportunities to test the adequacy of their current understandings. It is also useful to remember the educator's maxim. Teachers teach as they are taught, not as they are told to teach. Thus, trainers in constructivist professional development sessions model learning activities that teachers can apply in their own classrooms. It is not enough for trainers to describe new ways of teaching and expect teachers to translate from talk to action; it is more effective to engage teachers in activities that will lead to new actions in classrooms. Principles of learning from an information processing perspective such as recognizing the limits of short term memory, providing many opportunities for students to connect prior knowledge to current learning, and recognizing the need for space practice can also be implemented within a question forming approach. On the other hand, if I focus only on the desired end goals, especially knowledge goals, without consideration of the student's acquired knowledge and background, I run the risk of developing knowledge and skills that have no meaning to the learner and are therefore easily forgotten.

CONCLUSION:

Students got amazed at their own discovery. Educational change is a very complex phenomenon and is almost invariably driven by an interaction of diverse factors. It was not possible to identify or control these factors in the evaluation that underpins this report. Hence the results and tentative conclusions should be treated as suggestive rather than conclusive. They also know that all these new problems do not necessarily require new solution procedure but rather it is repetition of the same procedure which they already know. It helps establishing a base problem and looking for other problems that fit this pattern. These way students could easily manage problem and did not feel the burden of separate procedures. Similarly other areas of mathematics can be explored to see the effectiveness of ICT in handling the critical pedagogical issues.

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